

Commutative orders in semigroups

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We consider commutative orders, that is, commutative semigroups having a semigroup of quotients in a local sense defined as follows. An element $a \in S$ is *square-cancellable* if for all $x, y \in S^1$ we have that $xa^2 = ya^2$ implies $xa = ya$ and also $a^2x = a^2y$ implies $ax = ay$. It is clear that being square-cancellable is a necessary condition for an element to lie in a subgroup of an oversemigroup. In a commutative semigroup S , the square-cancellable elements constitute a subsemigroup $\mathcal{S}(S)$. Let S be a subsemigroup of a semigroup Q . Then S is a *left order* in Q and Q is a *semigroup of left quotients* of S if every $q \in Q$ can be written as $q = a^\sharp b$ where $a \in \mathcal{S}(S)$, $b \in S$ and a^\sharp is the inverse of a in a subgroup of Q and if, in addition, every square-cancellable element of S lies in a subgroup of Q . *Right orders* and *semigroups of right quotients* are defined dually. If S is both a left order and a right order in Q , then S is an *order* in Q and Q is a *semigroup of quotients* of S . We remark that if a commutative semigroup is a left order in Q , then Q is commutative so that S is an order in Q . A given commutative order S may have more than one semigroup of quotients. The semigroups of quotients of S are pre-ordered by the relation $Q \geq P$ if and only if there exists an onto homomorphism $\phi : Q \rightarrow P$ which restricts to the identity on S . Such a ϕ is referred to as an *S-homomorphism*; the classes of the associated equivalence relation are the *S-isomorphism classes of orders*, giving us a partially ordered set $\mathcal{Q}(S)$. In the best case, $\mathcal{Q}(S)$ contains maximum and minimum elements. In a commutative order S , $\mathcal{S}(S)$ is also an order and has a maximum semigroup of quotients R , which is a Clifford semigroup. We investigate how much of the relation between $\mathcal{S}(S)$ and its semigroups of quotients can be lifted to S and its semigroups of quotients.

This is a joint work with P. N. ÁNH, V. GOULD, AND P. A. GRILLET.