A note on extreme points of C^∞-smooth balls in polyhedral spaces

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Morris [Mo83] proved that every separable Banach space X that contains an isomorphic copy of c_0 has an equivalent strictly convex norm such that all points of its unit sphere S_X are unpreserved extreme, i.e., they are no longer extreme points of $B_{X^{**}}$. We use a result of Hájek [Ha95] to prove that any separable infinite-dimensional polyhedral Banach space has an equivalent C^∞-smooth and strictly convex norm with the same property as in Morris' result. We additionally show that no point on the sphere of a C^2-smooth equivalent norm on a polyhedral infinite-dimensional space can be strongly extreme, i.e., for any such x, there exists a sequence (h_n) in X with $\|h_n\| \not\to 0$ such that $\|x \pm h_n\| \to 1$. This is a joint work with A. J. Guirao (Valencia) and V. Zizler (Calgary).

References

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