

EIGENVALUES OF (r, p) -NUCLEAR OPERATORS AND APPROXIMATION PROPERTIES OF ORDER (r, p)

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This lecture may be considered as a small survey of results obtained very recently by me and my PhD student Qaisar Latif. We consider different types of nuclear operators and investigate the corresponding eigenvalues problems, applying the results for obtaining trace-formulas of Grothendieck–Lidskiĭ type. For more or less complete investigation of the problems, we had to introduce some new notions of approximation properties for Banach spaces with giving both positive and negative results in connection with the properties.

For example, we study the tensor products and corresponding operator ideals of the operators of kind

$(r; q)$ $T = \sum_{n=1}^{\infty} x'_n \otimes y_n$ with one of the sequence being weakly q -summable while another one is absolutely r -summable, for $r \in (0, 1]$ (so, T is nuclear).

For this cases, we have, in particular,

(i) if $q' = p \in [1, 2]$ and $T : X \rightarrow X$ is as above, then for $s > 0$ with $1/s = 1/r + 1/2 - 1/p$, T is of spectral type l_s . For $s = 1$, the nuclear trace and the spectral trace are well defined and coincide.

(ii) The result in (i) is sharp in a sense (roughly speaking, some approximation conditions are necessary). Discussing this sharpness, we give examples of operators U in Banach spaces, which are not nuclear, but whose adjoints U^* are of type mentioned in $(r; q)$ (even, for $2/3 < r < 1$).

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