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Traces of operators and their history

dedicated to the memory of Erhard Schmidt,
born on January 1, 1876, at Tartu

As every mathematician knows, the trace of a square matrix is defined to be the sum of all entries of the main diagonal. Extending this concept to the infinite-dimensional setting does not always work, since non-converging infinite series may occur. So one had to identify those operators that possess something like a trace. In a first step, this was done for operators on the separable Hilbert space. The situation in Banach spaces turned out to be much more complicated, as the missing approximation property causes a lot of trouble. I will present an axiomatic approach in which operator ideals play a dominant rule. My considerations include also singular traces that – by definition – vanish on all finite rank operators. Thanks to the discoveries of Connes, those traces became a useful tool in non-commutative geometry, in the theory of pseudo-differential operators, and in quantum mechanics. The lecture is intended to show that people who prefer to live in Hilbert spaces heavily need Banach spaces techniques.

REFERENCES:

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