

# SOME DUALITY RESULTS ON BOUNDED APPROXIMATION PROPERTIES OF PAIRS

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The bounded approximation property for pairs  $(X, Y)$  consisting of a Banach space  $X$  and a fixed subspace  $Y$  was recently introduced by Figiel, Johnson, and Pełczyński [FJP].

Johnson [J] proved that if the dual space  $X^*$  of a Banach space  $X$  has the bounded approximation property, then it also has the bounded duality approximation property. We extend Johnson's result to the pairs of Banach spaces as follows. *The pair  $(X^*, Y^\perp)$  has the bounded approximation property if and only if the pair  $(X, Y)$  has the bounded duality approximation property.*

We also present several reformulations of the bounded approximation property of pairs and its duality version, and study possibilities for lifting the bounded approximation properties of a pair  $(X, Y)$  to the pair  $(X^*, Y^\perp)$  in some special cases.

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## REFERENCES

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