

THE ASYMPTOTICALLY COMMUTING BOUNDED APPROXIMATION PROPERTY OF BANACH SPACES

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Our departure point is the following two theorems of Nigel J. Kalton et al. (see, e.g., [C, Theorems 4.6 and 9.3]):

1) separable Banach spaces with the metric approximation property (AP) have the commuting metric AP (Casazza–Kalton);

2) in the non-separable case, the commuting bounded AP implies the separable complementation property, hence, e.g., ℓ_∞ fails the commuting bounded AP (Casazza–Kalton–Wojtaszczyk).

We introduce and study a (strict) weakening of the commuting bounded AP – the asymptotically commuting bounded AP. It turns out that any dual space with the bounded AP also enjoys this property. Our principal result is that a Banach space with the asymptotically commuting bounded AP is saturated with separable closed subspaces having a nice form of the commuting bounded AP. These subspaces are locally complemented and, in some special cases, even complemented.

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REFERENCES

- [C] P. G. CASAZZA. *Approximation properties*. In: W.B. Johnson and J. Lindenstrauss (eds.) *Handbook of the Geometry of Banach Spaces*. Volume 1, Elsevier (2001), 271–316.

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