ON BLOCKS OF SKELETON TOLERANCES

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A tolerance relation of a lattice L is a reflexive and symmetric relation compatible with operations of the lattice. Let T be a tolerance on a lattice L and let $\emptyset \neq X \subseteq L$. If every two elements of X are in the relation T, then we call X a preblock of T. Blocks of T are maximal preblocks (with respect to inclusion). A tolerance T of a finite lattice L is called a glued tolerance if its transitive closure is the total relation L^2. The (unique) smallest glued tolerance of L is called the skeleton tolerance of L.

It is well known that every finite lattice is a skeleton of a finite distributive lattice [3] or, even more, of infinitely many finite distributive lattices [2]. However, we are going to show that there are lattices which cannot be blocks of the skeleton tolerance of any finite lattice. Of course, it is clear for distributive or modular lattices, especially, as it is known that their blocks of the skeleton tolerance are maximal boolean or, respectively, complemented (or, equivalently, atomistic) intervals of such lattices [1]. Although in the modular case blocks of the skeleton tolerance are H-irreducible (see, e.g., [2]), it is not true in the general case.

In the talk we provide a full characterization of lattices which can be blocks of the skeleton tolerance relation of a finite lattice. Moreover, we formulate a necessary condition for a lattice to be such a block in the case of finite distributive lattices with at most k-dimensional maximal boolean intervals.

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References

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