# HARDY INEQUALITIES AND INTERPOLATION OF LORENTZ SPACES ASSOCIATED TO A VECTOR MEASURE (I)

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It is well-known that if the classical Lions-Peetre real interpolation method $(\cdot,\cdot)_{\theta,q}$ $(0 < \theta < 1 \leq, q \leq \infty)$ is applied to a pair $(X, L^{\infty}(\mu))$, the result is the Lorentz space $L^{p,q}(\mu)$ with $p = \frac{1}{1-\theta}$, for every quasi-Banach space $X$ such that $L^1(\mu) \subseteq X \subseteq L^{1,\infty}(\mu)$ and any scalar positive measure $\mu$. See e.g. [2].

The aim of this communication is to extend this result in a twofold direction: From scalar measures $\mu$ to vector measures $m$, and from the classical real interpolation method $(\cdot,\cdot)_{\theta,q}$ to general real interpolation methods $(\cdot,\cdot)_{\rho,q}$ associated to parameter functions $\rho$. For parameter functions $\rho$ in certain classes of functions, these spaces $(X_0, X_1)_{\rho,q}$ were studied, first by Kalugina [4] and Gustavsson [3], and later by Persson [5], and other authors.

This extension procedure carries the necessity of introducing suitable Lorentz spaces $\Lambda_v^q(\|m\|)$ associated to a vector measure $m$ and a weight $v$ which fit with our interpolation spaces. As a consequence of our interpolation results, we will find conditions under which such spaces are actually normable quasi-Banach spaces.

Our approach is based on the relationship of the pair $(\rho, q)$ with the *Ariño-Muckenhoupt weights* (see [1] and [6]), and sheds light even to the scalar measure case, providing a different point of view for it.

## References


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