UNIFORMLY SQUARE BANACH SPACES

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We say that a Banach space is *locally uniformly square (LUS)* if for every x in the unit sphere S_X of X, there exists a sequence (y_n) in the unit ball B_X of X such that

$$\|x \pm y_n\| \to 1 \text{ and } \|y_n\| \to 1.$$

If X is LUS and the sequence (y_n) tends to 0 weakly, we say that X is *weakly uniformly square (ωUS)*.

We say that a Banach space is *uniformly square (US)* if for every $x_1, x_2, \dots, x_N \in S_X$, there exists a sequence (y_n) in B_X such that

$$\|x_i \pm y_n\| \to 1 \text{ for every } i = 1, \dots, N \text{ and } \|y_n\| \to 1.$$

The motivation for studying such spaces is the fact that they possess properties which in a sense are at the opposite side of the spectrum from the Radon-Nikodým property (any closed convex set has slices of arbitrarily small diameter). If X is

- LUS then the diameter of every slice of B_X is 2.
- ωUS then the diameter of every non-empty relatively weakly open subset of B_X is 2.
- US then the diameter of every finite convex combination of slices of B_X is 2.

Other basic properties of such spaces will be discussed as well.

References

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