NECKLACES AND Q-CYCLES

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Let $n \geq 2$ be a positive integer and q be a prime power. Consider necklaces consisting of n beads each of which has one of the given q colors. A primitive C_n-orbit is an equivalence class of n necklaces closed under rotation. A primitive C_n-orbits is self-complementary when it is closed under color matching. In [4], it is shown that the $1-1$ correspondence between the set of self-complementary primitive C_n-orbits and set of self-reciprocal irreducible monic (srim) polynomials.

Let N be positive integer with $\gcd(q, N) = 1$. A q-cycle(N) is a finite sequence of non-negative integers closed under multiplication by q. In [5], it is shown that q-cycles(N) are closely related to monic irreducible divisors of $x^N - 1$ in $\mathbb{F}_q[x]$.

Here we discuss the following:

(i) q-cycles(N) can be used to obtain information about srim-polynomials;

(ii) connection between q-cycles(N) and C_n-orbits;

(iii) alternative proof of Miller results mentioned above.

References

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