

# THE POSITIVITY PROBLEM FOR LINEAR RECURRENCE SEQUENCES

VICHIAN LAOHAKOSOL

A (real) linear recurrence sequence  $(u_n)_{n \geq 0}$  of order  $k \in \mathbb{N}, k \geq 2$ , is a sequence satisfying

$$(1) \quad u_n = a_1 u_{n-1} + a_2 u_{n-2} + \cdots + a_k u_{n-k} \quad (n \geq k),$$

where  $a_1, a_2, \dots, a_k$  ( $\neq 0$ ) and the initial values  $u_0, u_1, \dots, u_{k-1}$  are given real numbers. Two important decision problems related to linear recurrence sequences are

- The Skolem Problem: does a given linear recurrence sequence have a zero ?
- The Positivity Problem: are all the terms of a given linear recurrence sequence positive ?

It is known that the decidability of the Positivity Problem implies the decidability of the Skolem Problem. At present, the decidability of each of these problems remains open. However, certain partial results have already appeared, viz., the Positivity Problem for sequences satisfying a second order linear recurrence relation was shown to be decidable by Halava-Harju-Hirvensalo, [1], in 2006. The Positivity Problem for sequences satisfying a third or a fourth order linear recurrence relation was shown to be decidable by Laohakosol-Punnim-Tangsupphathawat, in [2], [3] and [4], respectively. Our objective is to discuss the decidability of the Positivity Problem, especially of low orders.

## REFERENCES

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DEPARTMENT OF MATHEMATICS, KASETSART UNIVERSITY, BANGKOK 10900,  
THAILAND

*E-mail address:* fscivil@ku.ac.th