APPROXIMATION IN THE WEIGHTED LEBESGUE SPACES

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Let $L^p(\Gamma)$ and $E^p(G)$ be the set of all measurable complex valued functions such that $|f|^p$ is Lebesgue integrable on Γ with respect to the arclength, and the Smirnov class of analytic functions in G respectively.

For $p > 1$, $L^p(\Gamma)$ and $E^p(G)$ are Banach spaces with respect to the norm

$$\| f \|_{E^p(G)} := \| f \|_{L^p(\Gamma)} := \left(\int_\Gamma | f(z) |^p | dz | \right)^{1/p}.$$

By υ we denote a weight function on Γ, i.e. $\upsilon : \Gamma \to [0, \infty]$, for which the set $\upsilon^{-1}(\{0, \infty\})$ has measure zero.

We assume that $\upsilon \in A_p(\Gamma)$, i.e. satisfies the well known Muckenhout condition on Γ.

With every weight $\upsilon \in A_p(\Gamma)$, $1 < p < \infty$,we associate the weighted Lebesgue spaces $L^p(\Gamma, \upsilon)$, consisting of all measurable functions f on Γ such that

$$\| f \|_{L^p(\Gamma,\upsilon)} := \left(\int_\Gamma | f(z) |^p \upsilon(z) | dz | \right)^{\frac{1}{p}} < \infty$$

and the weighted Smirnov spaces $E^p(G, \upsilon)$:

$$\| f \|_{E^p(G,\upsilon)} := \left\{ f \in E^1(G) : \| f \|_{L^p(\Gamma,\upsilon)} < \infty \right\}.$$

The spaces $L^p(\Gamma, \upsilon)$ and $E^p(G, \upsilon)$ become Banach spaces if $\upsilon \in A_p(\Gamma)$.

In this talk we discuss the direct problems of approximation theory by rational functions and by polynomials in the spaces $L^p(\Gamma, \upsilon)$ and $E^p(G, \upsilon)$, respectively. Some special cases of this problem were investigated in [1] and [2].

References

[1] D. M. ISRAFILOV, *Approximation by $p-$ Faber polynomials in the weighted Smirnov class $E^p(G, \omega)$ and the Bieberbach polynomials.* Constr. Approx., 17(2001), 335-351.

[2] D. M. ISRAFILOV, *Approximation by $p-$ Faber-Laurent rational functions in the weighted Lebesgue spaces.* Czechoslovak Mathematical Journal, 54(129)(2004), 751-765.

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