ON CONGRUENCE EXTENSION PROPERTY FOR ORDERED ALGEBRAS

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Let Ω be a type. An *ordered Ω-algebra* is a triplet $\mathcal{A} = (A, \Omega_A, \leq_A)$ comprising a poset $(A, \leq_A)$ and a set Ω_A of operations on A such that all the operations $\omega_A \in \Omega_A$ are monotone mappings. An *order-congruence* on $\mathcal{A}$ is an algebraic congruence θ on $\mathcal{A}$ such that $a\theta a'$ whenever

$$a \leq a_1 \theta a_2 \leq \ldots \leq a_{n-1} \theta a_n \leq a' \leq a'_1 \theta a'_2 \leq \ldots \leq a'_{m-1} \theta a'_m \leq a$$

for some $a_1, \ldots, a_n, a'_1, \ldots, a'_m \in A$. By a *precongruence* on $\mathcal{A}$ we mean a preorder on $\mathcal{A}$ which is compatible with operations and extends the order of $\mathcal{A}$.

We say that an ordered algebra $\mathcal{A}$ has the *congruence extension property* if every order-congruence θ on an arbitrary subalgebra $\mathcal{B}$ of $\mathcal{A}$ is induced by an order-congruence Θ on $\mathcal{A}$, i.e. $\Theta \cap (B \times B) = \theta$. We say that an ordered algebra $\mathcal{A}$ has the *precongruence extension property* if every precongruence σ on an arbitrary subalgebra $\mathcal{B}$ of $\mathcal{A}$ is induced by a precongruence Σ on $\mathcal{A}$.

We discuss some results and open problems about these two properties.

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