SURVEY OF VECTOR-VALUED ANALYTIC COMPOSITION OPERATORS

HANS-OLAV TYLLI

I will survey some recent results, due to a number of authors, and problems concerning vector-valued analytic composition operators first considered in [LST].

Let $\varphi : D \to D$ be an analytic map, where D is the unit disk in $\mathbf{C}$. As an extension of the classical scalar-valued theory of composition operators it is of interest to study the operators $f \mapsto C_\varphi(f) = f \circ \varphi$ on various Banach spaces consisting of vector-valued analytic functions $f : D \to X$, where X is an infinite dimensional complex Banach space. Relevant examples include the vector-valued Hardy spaces $H^p(X)$, Bergman spaces $A^p(X)$, spaces $BMOA(X)$ of functions of bounded mean oscillation, as well as weak versions of these spaces.

I will focus on properties illustrating some of the similarities, differences as well as new phenomena encountered compared to the classical scalar-valued setting. For instance, weak compactness is often the smallest relevant qualitative property, while one may also consider composition operators from weak to strong type spaces. Open problems motivated by the vector-valued setting will be stated.

References

[LST] P.D. Liu, E. Saksman and H-O. Tylli, *Small composition operators on analytic vector-valued function spaces*, Pacific J. M. **184** (1998), 295–310.

Department of Mathematics and Statistics, University of Helsinki, Finland

E-mail address: hojtylli@cc.helsinki.fi