AN OUTER MEASURE ON A COMMUTATIVE RING

DARIUSZ DUDZIK

Institute of Mathematics
Pedagogical University of Cracow
ul. Podchorążych 2, 30-084 Kraków
Poland
dariusz.dudzik@gmail.com

We propose a construction of an outer measure on a commutative ring R with identity, similar to standard constructions, but using prime ideals as covering sets.

Let $\mathcal{P} \subseteq \mathrm{Spec}(R)$ be a family of prime ideals such that

$$\bigcup \mathcal{P} = R \setminus R^{\times},$$

where $R^{\times}$ is the set of invertible elements of R, and let μ be a measure on $\mathcal{P}$. The outer measure $\mu^* : 2^R \longrightarrow [0, +\infty]$ induced by μ is defined by

$$\mu^*(A) = \inf_{\mathcal{S} \in \Omega(A)} \mu(\mathcal{S}),$$

where

$$\Omega(A) = \left\{ \mathcal{S} \subseteq \mathcal{P} : \mathcal{S} \text{ is } \mu\text{-measurable}, \bigcup \mathcal{S} \supseteq A \setminus R^{\times} \right\}.$$

In the talk, we will prove several properties of μ^* and provide a few examples related to algebraic geometry, functional analysis and number theory. (This is a joint work with Marcin Skrzyński).

REFERENCES

[1] M. F. Atiyah & I. G. MacDonald, *Introduction to Commutative Algebra*, Westview Press, 1994.
[2] H. Federer, *Geometric Measure Theory*, Springer, Berlin-Heidelberg-New York, 1969.
[3] W. Rudin, *Functional Analysis*, McGraw-Hill, 1991.