APPROXIMATION BY TRIGONOMETRIC BLACKMAN- AND ROGOSINSKI-TYPE OPERATORS

ANNA SAKSA, ANDI KIVINUKK

We study for the $2\pi-$periodic continuous functions, $f \in C_{2\pi}$, the trigonometric operators

$$U_n(f,x) = \frac{a_0}{2} + \sum_{k=1}^{n} \varphi(\frac{k}{n+1})(a_k \cos kx + b_k \sin kx),$$

where $\varphi \in C_{[0,1]}, \varphi(0) = 1, \varphi(1) = 0$.

Two type of operators will be defined:

1. $\varphi_a(t) = a + \frac{1}{2}\cos \pi t + (\frac{1}{2} - a)\cos 2\pi t, a \in \mathbf{R}$ defines the Blackman-type operators $B_{n,a} : C_{2\pi} \to C_{2\pi}$;

2. $\varphi_a(t) = a \cos \frac{\pi t}{2} + (1-a)\cos \frac{3\pi t}{2}, a \in \mathbf{R}$ defines the Rogosinski-type operators $R_{n,a} : C_{2\pi} \to C_{2\pi}$.

Theorem 1. *1. If $f \in C_{2\pi}$, then for any $a \in \boldsymbol{R}$*

$$\|B_{n,a}f - f\|_{C_{2\pi}} \le (\|B_{n,a}\| + |a| + \frac{1}{2} + |\frac{1}{2} - a|)E_n(f) + \\ + \frac{1}{4}\omega_2(f, \frac{\pi}{n+1}) + \frac{|1-2a|}{4}\omega_2(f, \frac{2\pi}{n+1}),$$

where $E_n(f)$ is the best approximation, and $\omega_2(f,\delta)$ is the 2-nd modulus of continuity.

2. In case $a = 5/8$ for any $f \in C_{2\pi}$ we have

$$\|B_{n,5/8}f - f\|_{C_{2\pi}} \le (\frac{5}{4} + \|B_{n,5/8}\|)E_n(f) + \frac{1}{16}\omega_4(f, \frac{\pi}{n+1}).$$

In some cases we are able to compute exact values of those operators norms $\|B_{n,a}\| = \sup\{\|B_{n,a}f\|_{C_{2\pi}} : \|f\|_{C_{2\pi}} \le 1\}$.

Theorem 2. *1. If $0 \le a \le 3/8$, then*

$$\sup_n \|B_{n,a}\| = (1-2a)(Sci(1) + Sci(4)) + 2a(Sci(2) + Sci(3)) = \\ = 1.064... - a(0.159...),$$

where $Sci(x) = Si(\pi x)/\pi$, $Si()$ - the integral sine.

2. In case if $a = 5/8$ we have $\sup_n \|B_{n,5/8}\| = 1.234...$

Analogous results are valid for the Rogosinski-type operators.

This research was partially supported by the Estonian Sci. Foundation, grant 8627.

Faculty of Mathematics, University of Tallinn, 10120 Tallinn, Narva road 25, Estonia

E-mail address: saksaanna@hot.ee, andik@tlu.ee