ERROR ESTIMATES FOR CARDINAL SPLINE INTERPOLATION AND QUASI-INTERPOLATION

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Denote by $S_{h,m} = \left\{ f_h \in C^{m-2} : f_h\Big|_{[ih,(i+1)h]} \in \mathcal{P}_{m-1},\ i \in \mathbb{Z} \right\}$ the space of (cardinal) splines of step size $h > 0$ and degree $m-1$, $m \in \mathbb{N}$. For $f \in BC(\mathbb{R})$, there exists a unique bounded (Wiener-Schoenberg) interpolant $Q_{h,m}f \in S_{h,m}$ interpolating f at the points $(k + \frac{m}{2})h$, $k \in \mathbb{Z}$. It holds [1]

$$\sup_{f \in W^{m,\infty}(\mathbb{R}),\ \|f^{(m)}\|_\infty = 1} \| f - Q_{h,m} f \|_\infty = \kappa_{m+1} \pi^{-m} h^m,$$

where $\kappa_m = \frac{4}{\pi} \sum_{k=0}^{\infty} (-1)^{km} (2k+1)^{-m}$ is the Favard constant. There exists no approximation method of higher accuracy provided that only the values $f((k + \frac{m}{2})h)$, $k \in \mathbb{Z}$, are exploited by the method.

For $m \geq 3$, computation of $(Q_{h,m}f)(x)$ at an intermediate point $x \in \mathbb{R}$ needs all values $f((k + \frac{m}{2})h)$, $k \in \mathbb{Z}$. We introduce a quasi-interpolant $Q'_{h,m}f \in S_{h,m}$, "almost" [2] preserving the accuracy of $Q_{h,m}f$ and such that $(Q'_{h,m}f)(x)$ needs the values of f only at $(k + \frac{m}{2})h \in [x - mh, x + mh]$.

Due to the last property, quasi-interpolation is preferable designing [3] fully discrete methods for integral equations; main ideas will be explained but we do not go into details in this talk.

REFERENCES

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