THE VARIATION DETRACTING PROPERTY OF SOME SHANNON SAMPLING SERIES AND THEIR DERIVATIVES

TARMO METSMÄGI

We consider the generalized Shannon sampling operators, which preserve the total variation of functions and their derivatives. The generalized Shannon sampling operators for the uniformly continuous and bounded functions on real line, $f \in C(\mathbb{R})$, are given by $(t \in \mathbb{R}; W > 0)$

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W}) s(Wt - k), \tag{1}$$

where kernel $s(t) := s(\lambda; t) := \int\limits_0^1 \lambda(u) \cos(\pi t u)\, du$, and $\lambda \in C_{[-1,1]}$ is an even window function, $\lambda(0) = 1$, $\lambda(u) = 0$ $(|u| \geqslant 1)$.

Proposition. *Define the related kernel to the kernel s as follows*

$$s_{m_1,\dots,m_n}(t) := \int_0^1 \frac{\lambda(u)}{\operatorname{sinc}(m_1 u) \dots \operatorname{sinc}(m_n u)} \cos(\pi t u) du \tag{2}$$

for $0 < m_1, \dots, m_n \leq 1$ *and* $\operatorname{sinc}(u) := \frac{\sin(\pi u)}{\pi u}$. *Then*

(3)

$$s(t) = \frac{1}{2^n m_1 \dots m_n} \int_{-m_1}^{m_1} dx_1 \int_{-m_2}^{m_2} dx_2 \dots \int_{-m_n}^{m_n} s_{m_1,\dots,m_n}(t + x_1 + \dots + x_n) dx_n.$$

Let $BV(\mathbb{R})$ denote the class of all functions of bounded variation on $\mathbb{R}$. The corresponding total variations are denoted by $V_{\mathbb{R}}[f]$. The variation detracting property for derivatives reads as follows.

Theorem. *Assume the kernel (2)* $s_{m_1,\dots,m_{n+1}} \in L^1(\mathbb{R})$ *for* $0 < m_1, \dots, m_{n+1} \leq 1$ *such that there exists* $b \in \mathbb{R}$ *with* $\pm m_1 \pm \dots \pm m_{n+1} - b \in \mathbb{Z}$. *If* f *is bounded and* $f^{(n)} \in BV(\mathbb{R})$, *then* $(S_W f)^{(n)} \in BV(\mathbb{R})$ *and*

$$V_{\mathbb{R}}[(S_W f)^{(n)}] \leq \|s_{m_1,\dots,m_{n+1}}\|_1 \, V_{\mathbb{R}}[f^{(n)}]. \tag{4}$$

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Institute of Mathematics and Natural Sciences, Tallinn University, 10120 Tallinn, 25 Narva Road, Estonia

E-mail address: `tmetsmag@tlu.ee`