ON STRONG AND STATISTICAL CONVERGENCES IN SOME FAMILIES OF SUMMABILITY METHODS

ANNA ŠELETSKI AND ANNE TALI

In our talk we discuss strong and statistical convergences in Cesàro- and Euler–Knopp-type families of summability methods.

Let $x = (x_n)$ be sequences with $x_n \in \mathbb{C}$ $(n = 0, 1, 2, ...)$ and A be a transformation which transforms a sequence x into the sequence $y = (y_n)$. If the limit $\lim_n y_n = s$ exists, then we say that x is convergent with respect to the summability method A (in short, A-convergent) to s. If $y_n = O(1)$, we say that x is bounded with respect to the method A (in short, A-bounded).

Let $\{A_\alpha\}$ be a Cesàro- or an Euler–Knopp-type family of summability methods A_α (see [4]), where α is a continuous parameter with values $\alpha > \alpha_0$ (α_0 is some fixed real number). Suppose that $[A_{\alpha+1}]_t$ are strong summability methods with positive exponents $t = (t_n)$ in the family $\{A_\alpha\}$ (see [3]).

We continue comparison of different strong summability methods $[A_{\alpha+1}]_t$ in the family $\{A_\alpha\}$, started in [3], with the help of a new convexity theorem. This convexity theorem gives sufficient conditions for the statement:

if $x = (x_n)$ is $[A_{\beta+1}]_t$-convergent to s and $[A_{\gamma+1}]_t$-bounded, then $x = (x_n)$ is $[A_{\delta+1}]_t$-convergent to s for any $\beta > \delta > \gamma > \alpha_0$ in a Cesàro-type family $\{A_\alpha\}$.

The theorem mentioned above can be seen as a generalization of some convexity theorems known earlier (see [5] for references).

Convergence of a sequence $x = (x_n)$ with respect to the different strong summability methods $[A_{\alpha+1}]_t$ in the family $\{A_\alpha\}$ is characterized also with the help of statistical convergence. Basing on papers [1] and [2] we compare A_α- and $[A_{\alpha+1}]_t$-convergences (and -boundnesses) of x with its A-statistical A_α-convergence (and, in particular, with its statistical A_α-convergence) for different values of parameter α, where A is some regular non-negative matrix method. All the results can be transferred to the particular cases of the family $\{A_\alpha\}$, e.g., to the families of some generalized Nörlund methods (N, p_n^α, q_n) (see [4]).

The research was partially supported by Estonian Targeted Financing Project SF140083s08.

References

[1] E. Kolk, *Matrix summability of statistically convergent sequences*, Analysis **13** (1993), 77–83.

[2] F. Móricz, *Tauberian conditions under which statistical convergence follows from statistical summability* $(C, 1)$, J. Math. Anal. Appl. **275** (2002), 277–287.

[3] V. Soomer and A. Tali, *On strong summability of sequences*, Acta Comment. Univ. Tartu. Math. **11** (2007), 57–68.

[4] U. Stadtmüller and A. Tali, *Comparison of certain summability methods by speeds of convergence*, Analysis Mathematica **29** (2003), 227–242.

[5] U. Stadtmüller and A. Tali, *Strong summability in certain families of summability methods*, Acta Sci. Math. (Szeged) **70** (2004), 639–657.

Institute of Cybernetics, Tallinn University of Technology, 12618 Tallinn, 21 Akadeemia tee, Estonia

E-mail address: annatar@ioc.ee

Institute of Mathematics and Natural Sciences, Tallinn University, 10120 Tallinn, 25 Narva mnt, Estonia

E-mail address: atali@tlu.ee