COMPATIBLE FUNCTION EXTENSION AND MAJORITY FUNCTIONS

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Let A be a set, $s \subseteq A^2$, $D \subseteq A^n$ and $f : D \to A$. We say that f preserves s if for every $\langle x_1, y_1 \rangle, \ldots, \langle x_n, y_n \rangle \in s$, whenever $\langle x_1, \ldots, x_n \rangle, \langle y_1, \ldots, y_n \rangle \in D$, then also $\langle f(x_1, \ldots, x_n), f(y_1, \ldots, y_n) \rangle \in s$. We say that f is S-compatible where S is a set of subsets of A^2 if it preserves all $s \in S$.

Recall that a ternary function f on a set A is called a *majority function* if $f(a, a, b) = f(a, b, a) = f(b, a, a) = a$ for all $a, b \in A$. The f is said to be a *Pixley function*, if $f(a, b, b) = f(a, b, a) = f(b, b, a) = a$ for all $a, b \in A$.

In [1] we proved, in particular, the following result.

Theorem. *Let A be a finite set and S be a sublattice of the lattice of all equivalence relations on A. Assume that S contains both the diagonal Δ_A and $\nabla_A = A^2$. Then the following are equivalent:*

(1) *for any positive integer n and any $D \subseteq A^n$, every S-compatible function $f : D \to A$ can be extended to an S-compatible function $A^n \to A$;*
(2) *there exists an S-compatible Pixley function on A.*

In the present work the following analog of this theorem has been obtained.

Theorem. *Let A be a finite set and S be a set of binary relations on A. Assume that S is closed with respect to forming intersections, relational products and converse relations, and contains both Δ_A and ∇_A. Then the following are equivalent:*

(1) *for any positive integer n and any $D \subseteq A^n$, every S-compatible function $f : D \to A$ can be extended to an S-compatible function $A^n \to A$;*
(2) *there exists an S-compatible majority function on A.*

References

1. Kaarli, K., Compatible function extension property, *Algebra Universalis*, 1983, **17**, 200–207.

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