ON $M(a, B, c)$-IDEALS IN BANACH SPACES

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We say that a closed subspace Y of a Banach space X is an *ideal satisfying the $M(a, B, c)$-inequality* (in short, an *$M(a, B, c)$-ideal*) in X if there is a norm one projection P on X^* such that $\ker P = Y^\perp$ and

$$\|ax^* + bPx^*\| + c\|Px^*\| \leq \|x^*\| \quad \forall b \in B, \forall x^* \in X^*.$$

This approach was first suggested by E. Oja and it allows us to handle well-known special cases of ideals, namely M-, h-, u- and $M(r, s)$-ideals (for definitions and references, see, e.g., [2]), in a more unified way.

We have developed easily verifiable equivalent conditions for a subspace of ℓ_∞^2 to be an $M(a, B, c)$-ideal.

Following what was done in [1] for $M(r, s)$-ideals, we obtain new results in a more general $M(a, B, c)$-setting. Our main results are as follows. Suppose X and Y are closed subspaces of a Banach space Z such that $X \subset Y \subset Z$. If X is an $M(a, B, c)$-ideal in Y and Y is an $M(d, E, f)$-ideal in Z, then X is an ideal satisfying a certain type of inequality in Z. Relying on this result, we show that if X is an $M(a, B, c)$-ideal in its second bidual, then X is an ideal satisfying a certain type of inequality in $X^{(2n)}$ for every $n \in \mathbb{N}$.

For illustration, we list here two corollaries of our results.

- *If X is an $M(a, B, c)$-ideal in Y and Y is an M-ideal in Z, then X is an $M(a, B, c)$-ideal in Z.*
- *If X is a u-ideal in X^{**}, then X is an $M\left(\frac{1}{2n-1}, \left\{-\frac{2}{2n-1}\right\}, 0\right)$-ideal in $X^{(2n)}$ for every $n \in \mathbb{N}$.*

References

[1] R. Haller, *On transitivity of $M(r, s)$-inequalities and geometry of higher duals of Banach spaces*, Acta Comment. Univ. Tartu. Math. **6** (2002), 9–13.

[2] E. Oja, *Geometry of Banach spaces having shrinking approximations of the identity*, Trans. Amer. Math. Soc. **352** (2000), 2801–2823.

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