ON $M(a,B,c)$-IDEALS IN BANACH SPACES

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We say that a closed subspace $Y$ of a Banach space $X$ is an *ideal satisfying the $M(a,B,c)$-inequality* (in short, an $M(a,B,c)$-ideal) in $X$ if there is a norm one projection $P$ on $X^*$ such that $\ker P = Y^\perp$ and

$$\|ax^* + bPx^*\| + c\|Px^*\| \leq \|x^*\| \ \forall b \in B, \forall x^* \in X^*.$$ 

This approach was first suggested by E. Oja and it allows us to handle well-known special cases of ideals, namely $M$, $h$, $u$- and $M(r,s)$-ideals (for definitions and references, see, e.g., [2]), in a more unified way.

We have developed easily verifiable equivalent conditions for a subspace of $\ell^\infty_2$ to be an $M(a,B,c)$-ideal.

Following what was done in [1] for $M(r,s)$-ideals, we obtain new results in a more general $M(a,B,c)$-setting. Our main results are as follows. Suppose $X$ and $Y$ are closed subspaces of a Banach space $Z$ such that $X \subset Y \subset Z$. If $X$ is an $M(a,B,c)$-ideal in $Y$ and $Y$ is an $M(d,E,f)$-ideal in $Z$, then $X$ is an ideal satisfying a certain type of inequality in $Z$. Relying on this result, we show that if $X$ is an $M(a,B,c)$-ideal in its second bidual, then $X$ is an ideal satisfying a certain type of inequality in $X^{(2n)}$ for every $n \in \mathbb{N}$.

For illustration, we list here two corollaries of our results.

- *If $X$ is an $M(a,B,c)$-ideal in $Y$ and $Y$ is an $M$-ideal in $Z$, then $X$ is an $M(a,B,c)$-ideal in $Z$.*
- *If $X$ is a $u$-ideal in $X^{**}$, then $X$ is an $M \left( \frac{1}{2n-1}, \left\{ -\frac{2}{2n-1} \right\}, 0 \right)$-ideal in $X^{(2n)}$ for every $n \in \mathbb{N}$.*

**REFERENCES**


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