COMMUTATIVE SUBALGEBRAS OF THE ALGEBRA OF SMOOTH OPERATORS

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In this talk we deal with the noncommutative Fréchet *-algebra $L(s', s)$ (so-called algebra of smooth operators) of continuous linear operators from the space

$$s' := \left\{ \xi = (\xi_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} : \exists q \in \mathbb{N}_0 \quad |\xi|_q' := \left(\sum_{j=1}^{\infty} |\xi_j|^2 j^{-2q} \right)^{\frac{1}{2}} < \infty \right\}$$

of slowly increasing sequences to the space

$$s := \left\{ \xi = (\xi_j)_{j\in\mathbb{N}} \in \mathbb{C}^{\mathbb{N}} : \forall q \in \mathbb{N}_0 \quad |\xi|_q := \left(\sum_{j=1}^{\infty} |\xi_j|^2 j^{2q} \right)^{\frac{1}{2}} < \infty \right\}$$

of rapidly decreasing sequences, where multiplication and involution are defined, in a natural way, through the composition of operators on the Hilbert space ℓ_2 and the hilbertian involution. The algebra $L(s', s)$ can be represented as the algebra of matrices $(a_{j,k})_{j,k\in\mathbb{N}}$ such that $\sup_{j,k\in\mathbb{N}} |a_{j,k}| j^q k^q < \infty$ for all $q \in \mathbb{N}_0$ (so-called rapidly decreasing matrices with matrix multiplication and matrix complex involution) and as the algebra $\mathcal{S}(\mathbb{R}^2)$ of Schwartz functions on $\mathbb{R}^2$ with the Volterra convolution $(f \cdot g)(x, y) := \int_{\mathbb{R}} f(x, z)g(z, y)\mathrm{d}z$ as multiplication and involution $f^*(x, y) := \overline{f(y, x)}$. In these forms, the algebra $L(s', s)$ usually appears and plays a significant role in papers on K-theory of Fréchet algebras, C^*-dynamical systems and in noncommutative geometry (see e.g. papers of Cuntz, Blackadar, Elliot, Natsume, Nest, Phillips).

We show that every closed commutative *-subalgebra of $L(s', s)$ is generated by a single (normal) operator and has a Schauder basis consisting of pairwise orthogonal projections. As a by-product we get a Hölder continuous functional calculus in $L(s', s)$. Next we show that every closed commutative *-subalgebra of $L(s', s)$ which is isomorphic (as a Fréchet space) to some complemented subspace of the space s is already isomorphic (as a Fréchet *-algebra) to some subalgebra of the algebra s (pointwise multiplication and involution). We also give an example of a closed commutative *-subalgebra of $L(s', s)$ which is not isomorphic to any subalgebra of s.

References

[1] Tomasz Ciaś, *On the algebra of smooth operators*. Preprint available at http://arxiv.org/abs/1304.7189, submitted to a journal.

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