ON SUMMABILITY OF ORTHOGONAL EXPANSIONS AND SHANNON SAMPLING SERIES

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Let X be a fixed Banach space, and we assume that any $f \in X$ is in some sense representable via an orthogonal expansion $f \sim \sum_{k=0}^{\infty} P_k f$, using a sequence of mutually orthogonal projections $\{P_k\}_{k=0}^{\infty}$. Often the convergence of that series will be succeeded using some summability methods (called as the (Φ, λ) method [1])

$$U(t)f = \sum_{k=0}^{\infty} \varphi(\lambda_k/t) P_k f, \quad t > 0. \tag{1}$$

If $\varphi(u) = (1-u^r)_+,\ r \in \mathbf{N}$, then we get the Zygmund method, denoting it by $Z_n^r,\ n \in \mathbf{N}$. Often it forms an approximation process with known order of approximation, i.e. $\|f - Z_n^r f\| \le M_r \Omega_r(f, 1/n) \to 0,\ n \to \infty$, where $\Omega_r(f, 1/n)$ is a modulus of continuity, a K-functional or another known sequence.

We will demonstrate that some quite general operators (1) can be considered using a subordination equality

$$f - U_n f = \sum_{j=r}^{\infty} c_j (f - Z_n^r f), \tag{2}$$

under essential assumption $\varphi(u) = 1 - \sum_{j=r}^{\infty} c_j u^j$. Our application of (2) go to the Shannon sampling series [2].

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References

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