MULTIPLE FOURIER EXPANSIONS OVER WALSH-PALEY AND TRIGONOMETRIC SYSTEMS

SVETLANA BLOSHANSKAYA, IGOR BLOSHANSKII, AND OLGA LIFANTSEVA

We consider $\mathbb{I}^N = [0,1)^N$, $N \geq 2$, and two orthonormal on $\mathbb{I}^N$ sytems $\Psi = \mathcal{E}$ and $\Psi = W$ (where $\mathcal{E} = \{e^{i2\pi nx}\}_{n\in\mathbb{Z}^N}$, is the multiple trigonometric system, and $W = \{w_n(x)\}_{n\in\mathbb{Z}_0^N}$, is the multiple Walsh-Paley system, $x \in \mathbb{I}^N$, $\mathbb{Z}_0^N = \{n \in \mathbb{Z}^N : n_j \geq 0,\ j = 1,\dots,N\}$).

Let E be an arbitrary measurable set, $E \subset \mathbb{I}^N$, $\mu E > 0$ ($\mu = \mu_N$ is N-dimensional Lebesgue measure), and let $\mathcal{A} = \mathcal{A}(\mathbb{I}^N)$ be some linear subspace of $L_1(\mathbb{I}^N)$.

We investigate the behavior of rectangular partial sums $S_n(x,f;\Psi)$ of multiple Fourier series over the system Ψ (here $x \in \mathbb{I}^N$, $f \in \mathcal{A}(\mathbb{I}^N)$, the vector $n \in \mathbb{Z}_0^N$) as $n \to \infty$, i.e. $\min\limits_{1\leq j\leq N} n_j \to \infty$, on the sets E and $\mathbb{I}^N \setminus E$ depending: on the smoothness of the function f (i.e. on the type of the space $\mathcal{A}$), on structural and geometric characteristics of the set E (SGC(E)), as well as on restrictions imposed on the components $n_1,\dots,n_N$ of the vector n (the "index" of the partial sum $S_n(x,f;\Psi)$).

In [1] for the wide class of measurable sets $\{E\}$, $E \subset \mathbb{I}^N$, $N \geq 3$, we have obtained a criterion for validity (in the terms of SGC(E)) of weak generalized localization almost everywhere of the considered Fourier series (i.e. the necessary and sufficient conditions for convergence almost everywhere on some set E_1, $E_1 \subset E$, $\mu E_1 > 0$, of the considered series, when the expanded function equals zero on E), in the case $\mathcal{A}(\mathbb{I}^N) = L_p(\mathbb{I}^N)$, $p > 1$, and partial sums $S_n(x,f;\Psi)$ have "index" $n = (n_1,\dots,n_N)$, in which some of the components are elements of (single) lacunary sequences (i.e. for some components n_j of the vector n the following conditions are satisfied $\frac{n_j^{(s+1)}}{n_j^{(s)}} \geq q > 1, s = 1,2,\dots$).

The problem indicated above was investigated by us in the Orlicz spaces as well. We have also shown how the results formulated above can be "sewed" with the results obtained earlier in [2, 3].

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References

[1] S.K.Bloshanskaya, I.L.Bloshanskii, O.V.Lifantseva, *Trigonometric Fourier series and Walsh-Fourier series with lacunary sequence of partial sums,*

Matem. zametki, **93**:2 (2013), 305–309; English transl. in Mathematical Notes, **93**:2 (2013), 332-336.

[2] I.L.Bloshanskii, *On criteria for weak generalized localization in N-dimensional space*, Dokl. Akad. Nauk SSSR, **271**:6 (1983), 1294–1298; English transl. in Soviet Math. Dokl., **28** (1983).

[3] S.K.Bloshanskaya, I.L.Bloshanskii, *Weak generalized localization for multiple Walsh-Fourier series of functions in $L_p, p \geq 1$*, Trudy Mat. Inst. Steklov, **214** (1997), 83–106.

Department of Higher Mathematics, National Research Nuclear University, 115409 Moscow, 31 Kashirskoye Shosse, Russia
E-mail address: ig.bloshn@gmail.com

Department of Mathematical Analysis and Geometry, Moscow State Regional University, 105005 Moscow, 10a Radio Street, Russia
E-mail address: ig.bloshn@gmail.com, ov-lifantseva@yandex.ru