ON APPROXIMATION PROPERTIES OF KANTOROVICH-TYPE SAMPLING OPERATORS

OLGA ORLOVA, GERT TAMBERG

For the uniformly continuous and bounded functions $f \in C(\mathbb{R})$ the generalized sampling operators S_W and the corresponding Kantorovich-type sampling operators $S^K_{W,n}$ (cf. [1]) ($n \in \mathbb{N}$) are given by ($t \in \mathbb{R}$; $W > 0$)

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(\frac{k}{W}) s(Wt - k),$$

$$(S^K_{W,n} f)(t) := \sum_{k=-\infty}^{\infty} \left(nW \int\limits_{(2nk-1)/2nW}^{(2nk+1)/2nW} f(u)\, du \right) s(Wt - k).$$

Since in many applications the results of measurements are some local averages, not the exact point estimates, it is more natural to consider Kantorovich-type sampling operators $S^K_{W,n}$ instead of operators S_W.

We show, that we can use the results we have for operators S_W to prove analogous results for operators $S^K_{W,n}$.

Theorem 1. *If the sampling operator* $S_W : C(\mathbb{R}) \to C(\mathbb{R})$ *has the finite norm, i.e.*

$$\|S_W\| = \sup_{u \in \mathbb{R}} \sum_{k=-\infty}^{\infty} |s(u - k)| < \infty,$$

then the corresponding Kantorovich-type operator $S^K_{W,n}$ *has the norm* $\|S^K_{W,n}\| = \|S_W\|$ $(n \in \mathbb{N})$.

Theorem 2. *If we can estimate the order of approximation by the operator* S_W *via the modulus of smoothness of order* $r \geqslant 2$*, then we have for the corresponding Kantorovich-type operator* $S^K_{W,n}$ *the estimate*

$$\|S^K_{W,n} f - f\|_C \leqslant M\omega_2(f, 1/W).$$

This research was supported by the Estonian Science Foundation, grant 9383, and by the Estonian Min. of Educ. and Research, project SF0140011s09.

References

[1] C. Bardaro, G. Vinti, P. L. Butzer, R. L. Stens, *Kantorovich-type generalized sampling series in the setting of Orlicz spaces*, Sampling Theory in Signal and Image Processing **6** (2007), 29–52.

Dept. of Mathematics, Tallinn University of Technology, 19086 Tallinn, 5 Ehitajate tee, Estonia

E-mail address: olga.orlova@artun.ee, gtamberg@staff.ttu.ee