TWO APPROXIMATION PROPERTIES

ANATOLIJ PLICHKO

We consider the following approximation properties.

The first property takes origin in Numerical Mathematics, see e.g. [1]. A Banach space X has *norm approximation property* if there is $\lambda \geq 1$ such that for every $\varepsilon > 0$ and every finite-dimensional subspace $E \subset X$ there is a finite dimensional operator $T : X \to X$ with $\|T\| \leq \lambda$ and such that for all $x \in E$

$$(1-\varepsilon)\|x\| \leq \|Tx\| \leq (1+\varepsilon)\|x\|.$$

A Banach space X has *bounded separable approximation property* (see e.g. [2]) if there exists $\lambda \geq 1$ such that for every finite set $F \subset X$ there is a separable rank operator $T : X \to X$ with $\|T\| \leq \lambda$ and $Tx = x$ for each $x \in F$.

We show that the well known Pisier space has not the norm approximation property. We know no Banach space without bounded separable approximation property.

References

[1] G. Vainikko, *Funktionalanalysis der Diskretisierungsmethoden*, Teubner-Texte zur Mathematik. B.G. Teubner Verlag, Leipzig, 1976.

[2] S. Berrios, G. Botelho, *Approximation properties determined by operator ideals and approximability of homogeneous polynomials and holomorphic functions*, Studia Math. **208** (2012) 97–116.

Department of Mathematics, Cracow University of Technology, 31-155 Cracow, 24 Warszawska Street, Poland

E-mail address: aplichko@pk.edu.pl