ON THE BEST APPROXIMATION OF FUNCTIONS FROM HÖLDER CLASSES BY SUBSET OF LINEAR FINITE-RANK POSITIVE METHODS

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Let C be the space of continuous on $[0,1]$ functions endowed with standard norm $\|f\| = \max\{|f(x)| : x \in [0,1]\}$, and let ω be a concave modulus of continuity, i.e. concave, non-decreasing function satisfying $\omega(0) = 0$. By H^ω denote the class of functions $f \in C$ such that $|f(x) - f(y)| \leqslant \omega(|x - y|)$, $x, y \in [0,1]$. In particular, when $\omega(t) = t^\alpha$, $\alpha \in (0,1]$, classes H^ω are called *the Hölder classes* and are denoted H^α.

For $N \in \mathbb{N}$, let $\mathcal{L}_N$ be the set of all linear continuous N-rank operators $A : C \to C$. We consider the problem of the best approximation of functions from H^ω by methods from some subset $\mathcal{M} \subset \mathcal{L}_N$. To this end we define

$$\lambda_N\left(H^\omega; \mathcal{M}\right) := \inf_{A \in \mathcal{N}} \sup_{f \in H^\omega} \|f - Af\|.$$

Remark that the quantity $\lambda_N\left(H^\omega; \mathcal{L}_N\right)$ is called *the linear width* of class H^ω in space C. Question of its evaluation remains open and was repeatedly posed by N.P. Korneichuk (see, for instance, [1]). We solve this problem for $N = 1$.

In addition, we find $\lambda_N\left(H^\omega; \mathcal{M}\right)$ for the class $\mathcal{M} = \mathcal{L}_N^{++}$ of linear continuous N-rank operators A that can be represented in the form

$$Af = e_1 \int_0^1 f(t)\, dg_1(t) + \ldots + e_N \int_0^1 f(t)\, dg_N(t), \quad f \in C,$$

with some non-negative continuous functions $e_1, e_2, \ldots, e_N$ and non-decreasing functions $g_1, g_2, \ldots, g_N$. In particular, we obtain

$$\lambda_N\left(H^\alpha; \mathcal{L}_N^{++}\right) = \frac{\lambda_1\left(H^\alpha; \mathcal{L}_1\right)}{N^\alpha} = \frac{\Gamma(2-\alpha)\Gamma\left(\frac{1}{2} + \frac{\alpha}{2}\right)}{2N^\alpha \Gamma\left(\frac{3}{2} - \frac{\alpha}{2}\right)}.$$

References

[1] N. P. Korneichuk, *Extreme values of functionals and best approximation on classes of periodic functions*, Math. of USSR-Izvestiya. **5** (1971), 97–129.

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