

# LIPSCHITZ FUNCTIONS AND $M$ -IDEALS

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In [BW], Heiko Berninger and Dirk Werner tried to answer the following question.

**Berninger–Werner problem.** *Is the little Hölder space  $\text{lip}([0, 1]^\alpha)$  an  $M$ -ideal in the Hölder space  $\text{Lip}([0, 1]^\alpha)$  for every  $\alpha \in (0, 1)$ ?*

They showed that  $\text{lip}([0, 1]^\alpha)$  is an  $M$ -ideal in a non-separable subspace of  $\text{Lip}([0, 1]^\alpha)$ , whilst they conjectured that the answer to the problem might be negative.

In [K, Theorem 6.6], Nigel J. Kalton proved that for a compact metric space  $M$  and every  $\epsilon > 0$ , the little Lipschitz space  $\text{lip}(M)$  is  $(1+\epsilon)$ -isomorphic to a subspace of  $c_0$ .

As it is well-known that  $\text{lip}(M^\alpha)$  is a canonical predual of  $\mathcal{F}(M^\alpha)$  for a compact metric space  $M$ , Kalton solved Berninger–Werner problem in full generality, which means that the following holds: *if  $M$  is a compact metric space, then the little Hölder space  $\text{lip}(M^\alpha)$  is an  $M$ -ideal in the Hölder space  $\text{Lip}(M^\alpha)$ .*

We show how to use Kalton’s theorem to prove some results concerning properties of spaces  $\text{lip}(M)$ ,  $\mathcal{K}(\text{lip}(M))$  and  $\mathcal{L}(\text{lip}(M))$  for compact metric space  $M$ . We also present some further applications.

## REFERENCES

- [BW] HEIKO BERNINGER, DIRK WERNER, *Lipschitz spaces and  $M$ -ideals*, Extracta Math. **18** (2003), 33–56.
- [K] NIGEL J. KALTON, *Spaces of Lipschitz and Hölder functions and their applications*, Collect. Math. **55**, 2 (2004), 171–217.

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