MATRIX IDENTITIES INVOLVING MULTIPLICATION AND TRANSPOSITION

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Matrices and matrix operations constitute basic tools for algebra and analysis. Important properties of matrix operations are often expressed in form of *laws* or *identities* such as the associative law for multiplication of matrices. Studying matrix identities that involve multiplication and addition is a classic research direction motivated by several important problems in geometry and algebra. Matrix identities involving along with multiplication and addition also certain involution operations (such as taking the usual or symplectic transpose of a matrix) have attracted much attention as well.

If one aims to classify matrix identities of a certain type, then a natural approach is to look for a collection of 'basic' identities such that all other identities would follow from these basic identities. Such a collection is usually referred to as a *basis*. For instance, all identities of matrices over an infinite field involving only multiplication are known to follow from the associative law. Thus, the associative law forms a basis of such 'multiplicative' identities. For identities of matrices over a finite field or a field of characteristic 0 involving both multiplication and addition, the powerful results by Kruse–L'vov and Kemer ensure the existence of a finite basis. In contrast, multiplicative identities of matrices over a finite field admit no finite basis.

Here we consider matrix identities involving multiplication and one or two natural one-place operations such as taking various transposes or Moore-Penrose inversion. Our results may be summarized as follows.

None of following sets of matrix identities admit a finite basis:

- the identities of $n \times n$-matrices over a finite field involving multiplication and usual transposition;
- the identities of $2n \times 2n$-matrices over a finite field involving multiplication and symplectic transposition;
- the identities of 2×2-matrices over the field of complex numbers involving either multiplication and Moore-Penrose inversion or multiplication, Moore-Penrose inversion and Hermitian conjugation;
- the identities of Boolean $n \times n$-matrices involving multiplication and transposition.

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